Exercise 48

The number N of locations of a popular coffeehouse chain is given in the table. (The numbers of locations as of October 1 are given.)

Year	2004	2006	2008	2010	2012
N	8569	12,440	16,680	16,858	18,066

(a) Find the average rate of growth

(i) from 2006 to 2008

(ii) from 2008 to 2010

In each case, include the units. What can you conclude?

(b) Estimate the instantaneous rate of growth in 2010 by taking the average of two average rates of change. What are its units?

(c) Estimate the instantaneous rate of growth in 2010 by measuring the slope of a tangent.

Solution

Part (a)

Calculate the average rate of growth over each of the time intervals.

$$\text{(i)}\quad [2006, 2008]: \qquad \frac{N(2008) - N(2006)}{2008 - 2006} = \frac{16{,}680 - 12{,}440}{2} = 2120\ \frac{\text{locations}}{\text{year}}$$

$$\text{(ii)}\quad [2008, 2010]: \qquad \frac{N(2010) - N(2008)}{2010 - 2008} = \frac{16{,}858 - 16{,}680}{2} = 89\ \frac{\text{locations}}{\text{year}}$$

The coffeehouse chain added thousands of new locations per year on average from 2006 to 2008. From 2008 to 2010, however, the chain was shy of adding 90 new locations per year on average.

Part (b)

Calculate the average rate of growth over $[2010, 2012]$.

$$\text{(iii)}\quad [2010, 2012]: \qquad \frac{N(2012) - N(2010)}{2012 - 2010} = \frac{18{,}066 - 16{,}858}{2} = 604\ \frac{\text{locations}}{\text{year}}$$

For the best estimate of the instantaneous rate of change at $t = 2010$, take the average of the average rates taken over $[2008, 2010]$ and $[2010, 2012]$, the smallest time intervals about $t = 2010$.

$$\frac{89 + 604}{2} = 346.5\ \frac{\text{locations}}{\text{year}}$$